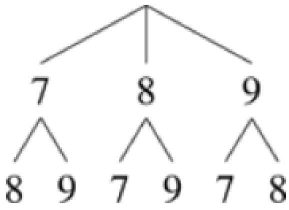
Exercise 5A: Solutions

1 We multiply the number of ways of making each choice. Therefore, there are $5 \times 3 \times 3 = 45$ different outfits.

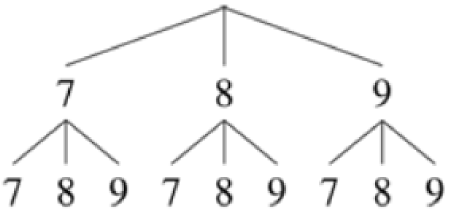
2 As a customer cannot select both chicken and beef, the total number of meals is $5 + 3 = 8$.

3 Each of the ten boys shakes hands with twelve girls so there are $10 \times 12 = 120$ handshakes.

4 **a**



b



5 **a** There are 3 choices for each of the 3 digits so $3 \times 3 \times 3 = 27$ three-digit numbers can be formed.

b For the first digit there are 3 choices. For the second there are 2 choices. For the final digit there is only 1 choice. Therefore, $3 \times 2 \times 1 = 6$ three digit numbers can be formed.

6 There are $4 + 2 = 6$ ways to get from Sydney to Adelaide and $2 + 3 = 5$ ways to get from Adelaide to Perth. Therefore, there is a total of $6 \times 5 = 30$ ways to get from Sydney to Perth.

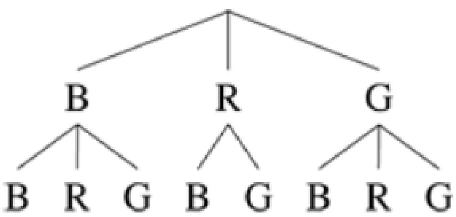
7 **a** $2 \times 3 = 6$

b $3 \times 2 \times 3 = 18$

c
$$\begin{aligned} &(1 \times 3 \times 2 \times 2) + (2 \times 2 \times 2 \times 1) \\ =&12 + 8 \\ =&20 \end{aligned}$$

d
$$\begin{aligned} &3 \times (2 \times 2 + 1) \\ =&3 \times 5 \\ =&15 \end{aligned}$$

8

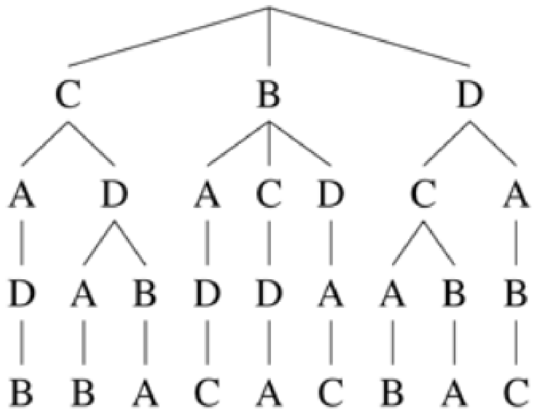


Reading off the tree diagram, the different arrangements are BB,BR,BG,RB,RG,GB,GR,GG.

9 Start by considering coins of the highest value and successively replace these with coins of lower values. This gives 12 ways of making change for 50 cents:

$$(20, 20, 10)$$
$$(20, 20, 5, 5)$$
$$(20, 10, 10, 10)$$
$$(20, 10, 10, 5, 5)$$
$$(20, 10, 5, 5, 5, 5)$$
$$(20, 5, 5, 5, 5, 5, 5)$$
$$(10, 10, 10, 10, 10)$$
$$(10, 10, 10, 10, 5, 5)$$
$$(10, 10, 10, 5, 5, 5, 5)$$
$$(10, 10, 5, 5, 5, 5, 5, 5)$$
$$(10, 5, 5, 5, 5, 5, 5, 5, 5)$$
$$(5, 5, 5, 5, 5, 5, 5, 5, 5, 5)$$

10 Let us denote the desks belonging to the first, second, third and fourth teacher by A, B, C and D respectively. When the teachers select new desks this corresponds to a rearrangement of the letters ABCD. We are interested in the rearrangements in which no letter is in its original position. We illustrate the possibilities on a tree diagram.



This gives 9 possible arrangements.

11a There are three choices for first position, two for the second and one for the third. This gives a total of $3 \times 2 \times 1 = 6$ arrangements.

b Call the runners A, B, C. Without a tie there are 6 arrangements. If there is two-way tie for first then either (A, B), (A, C) or (B, C) tie. This gives 3 possibilities. If there is a two-way tie for second then there are also 3 possibilities. Finally, if there is a three-way tie for first then there is only one possibility. This gives a total of $6 + 3 + 3 + 1 = 13$ different arrangements.

12 There are more efficient strategies but the easiest method is to count the number of unique entries in the table below.

$\times$	0	2	3	5	7	11
0	0	0	0	0	0	0
2	0	4	6	10	14	22
3	0	6	9	15	21	33
5	0	10	15	25	35	55
7	0	14	21	35	49	77
11	0	22	33	55	77	121

There are 16 unique entries in this table.